

# ***Adsorption Modeling Update***

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## *Theory / Practice*

### **Theory**

Improve our understanding of high-pressure adsorption equilibria through rigorous methodologies.

### **Practice**

Provide reliable models for optimum production of coalbed methane and CO<sub>2</sub> sequestration.

### **Goal**

Develop reliable coal-structure-based generalized equilibrium models using simple, accessible characterizations.

### **Strategy**

Use rigorous methodologies rooted in fundamentals to develop reliable methods of high industrial utility.

## ***Equilibrium Adsorption Modeling***

- We seek simple, reliable adsorption equilibrium models that are suitable for generalized predictions and reservoir simulations.
- Such models should:
  - Precisely represent pure and mixture isotherms
  - Facilitate accurate *a priori* predictions

# ***Current Issues in Equilibrium CBM Adsorption Modeling***

## ***Fundamental Theory - OSU***

- Improved mixture modeling 2002
- Computational algorithms 2002
- Effect of moisture on adsorption behavior 2003
- Model parameter generalizations 2003

## ***Matrix Characterization - PSU***

- Coal characterization 2003/4
- Coal-structure-based generalizations 2003/4

## *Current Issues in Equilibrium CBM Adsorption Modeling - 2*

### Experimental Work

- Effect of moisture on modeling behavior 2003
- Estimates for adsorbed-phase density
- Matrix swelling
- Near-critical behavior

### Reservoir Simulations

- Balancing computational efficiency and reliability
- Reservoir model calibrations

# Improved Mixture Adsorption Modeling

## ***Equilibrium Modeling: Four Methods***

1. Enhanced forms of Langmuir isotherms
  - provide simple data correlation
2. Two-dimensional equations of state (2-D EOS)
  - facilitate generalized simulations
3. Simplified-Local-Density (SLD-EOS) models
  - account for surface structure and near-critical behavior
4. The Ono-Kondo (OK) lattice model
  - provides a layering analogue for adsorption

## Objectives

The equilibrium models considered have been successful in modeling pure adsorption isotherms. Our objectives are to:

- Amend the one-fluid mixing rules and develop alternative mixing rules to improve **2-D EOS** mixture predictions.
- Extend the **SLD-EOS** and **OK** lattice models to high-pressure mixture adsorption modeling.



# Improved 2-D EOS Mixture Modeling

## *Hypotheses for Improved Mixing*

We have investigated two new approaches for improving the 2-D EOS mixing rules, and consequently, the 2-D EOS mixture predictions.

- First, we hypothesized that different molecules have different accessible adsorption surface areas.
- Second, we incorporated excess Gibbs free energy mixing rules in the 2-D EOS.

## Accessible Surface Area

- 2-D EOS (OSU, 1992):

$$\left[ \pi + \frac{a\sigma^2}{1 + Ub\sigma + W(b\sigma)^2} \right] [1 - (b\sigma)^m] = \sigma RT$$

- Original mixing and combination rules:

$$a = \sum_i \sum_j x_i x_j a_{ij}$$

$$b = \sum_i \sum_j x_i x_j b_{ij}$$

$$a_{ij} = (a_i + a_j)(1 - C_{ij})/2 \quad b_{ij} = (b_i + b_j)(1 + D_{ij})/2$$

- Relations between the parameters:

$$\alpha = a/A \quad \beta = b/A$$

## Accessible Surface Area Effect

Different adsorbates may have different accessible surface area on the same adsorbent surface.

$$A\alpha = \sum_i \sum_j x_i x_j a_{ij} = x_1^2 A_1 \alpha_1 + x_1 x_2 (\alpha_1 A_1 + \alpha_2 A_2)(1 - C_{ij}) + x_2^2 A_2 \alpha_2$$

Where:

$$A = \sum_i \sum_j x_i x_j A_{ij} = x_1^2 A_1 + x_1 x_2 (A_1 + A_2)(1 - L_{ij}) + x_2^2 A_2$$

$$E_{21} = A_2 / A_1 \quad \tilde{A} = A / A_1$$

$$\alpha = (x_1^2 \alpha_1 + x_1 x_2 (\alpha_1 + \alpha_2 E_{21})(1 - C_{ij}) + x_2^2 \alpha_2 E_{21}) / \tilde{A}$$

$$\beta = (x_1^2 \beta_1 + x_1 x_2 (\beta_1 + \beta_2 E_{21})(1 + D_{ij}) + x_2^2 \beta_2 E_{21}) / \tilde{A}$$

## Wong-Sandler Mixing Rules for 3-D Cubic Equation of State (Sandler, 1992)

$$b - \frac{a}{RT} = \sum_i \sum_j x_i x_j \left( b - \frac{a}{RT} \right)_{ij}$$

$$\frac{A_\gamma^{\text{ex}}}{CRT} = -\frac{a}{bRT} + \sum_i x_i \frac{a_i}{b_i RT}$$

where C is a constant and the cross term is:

$$\left( b - \frac{a}{RT} \right)_{ij} = \frac{1}{2} \left[ \left( b_i - \frac{a_i}{RT} \right) + \left( b_j - \frac{a_j}{RT} \right) \right] (1 - K_{ij})$$

## Wong-Sandler Mixing Rules for 2-D EOS

The Wong-Sandler mixing rules are extended to 2-D EOS:

$$b = \beta A \qquad a = \alpha A$$

$$\beta - \frac{\alpha}{RT} = \left( \sum_i \sum_j x_i x_j \left( \beta - \frac{\alpha}{RT} \right)_{ij} \right) / \tilde{A}$$

$$\left( \beta - \frac{\alpha}{RT} \right)_{ij} = \frac{1}{2} \left[ \left( \beta_i - \frac{\alpha_i}{RT} \right) E_i + \left( \beta_j - \frac{\alpha_j}{RT} \right) E_j \right] (1 - C_{ij})$$

## *Experimental Data*

- In the evaluations presented here, we used our gas adsorption measurements on dry activated carbon, specifically:
  - Adsorption isotherms for pure methane, nitrogen, CO<sub>2</sub> and their binary mixtures at 318.2 K (113 °F) and pressures to 12.4 MPa (1800 psia)
  - Nominal gas feed compositions of 20, 40, 60, and 80%

## Case Studies

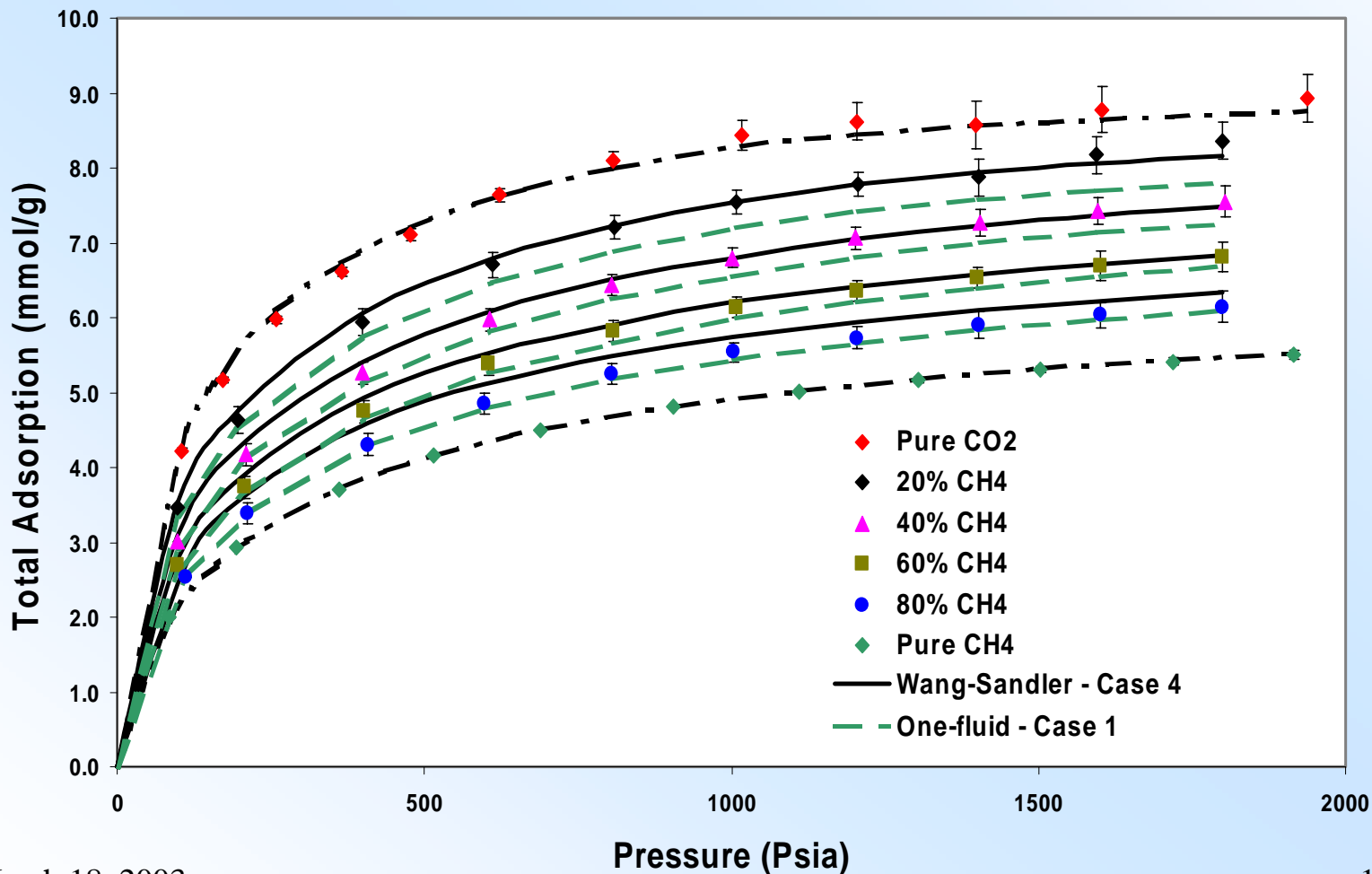
Case 1: One-fluid mixing rules -  $C_{12}$  and  $D_{12}$  are regressed

Case 2: Accessible area, one-fluid mixing rules - only surface area ratio,  $E_{21}$ , is regressed

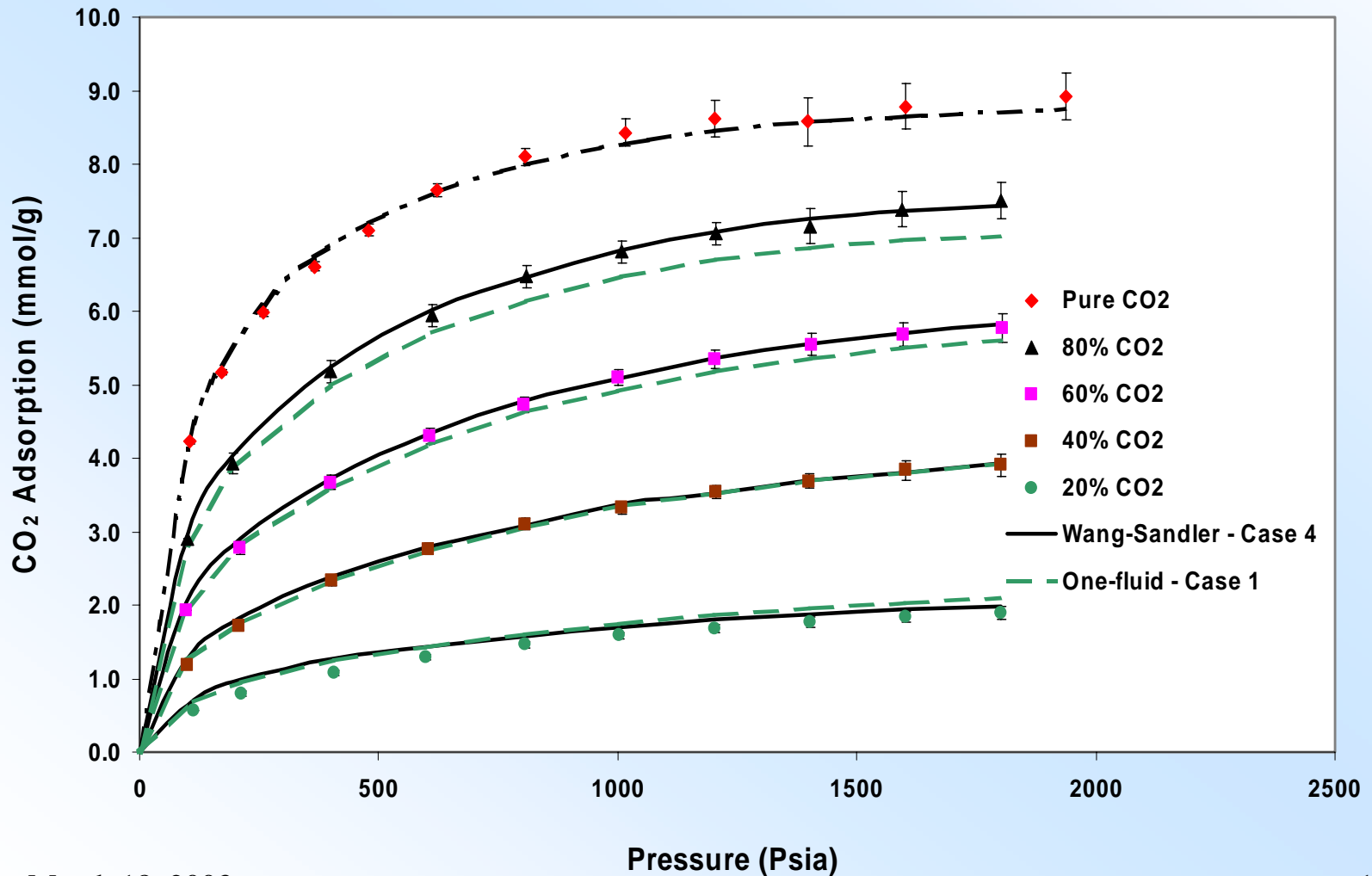
Case 3: Accessible area, one-fluid mixing rules -  $C_{12}$ ,  $D_{12}$ ,  $E_{21}$ , and surface interaction parameter,  $L_{12}$ , are regressed

Case 4: Wong-Sandler mixing rules with NRTL model -  $C_{ij}$ , and NRTL parameters,  $\tau_{12}$ ,  $\tau_{21}$  and  $\alpha_{12}$ , are regressed

## Total Absolute Adsorption for CH<sub>4</sub>/CO<sub>2</sub> System



## CO<sub>2</sub> Absolute Adsorption for CH<sub>4</sub>/CO<sub>2</sub> System

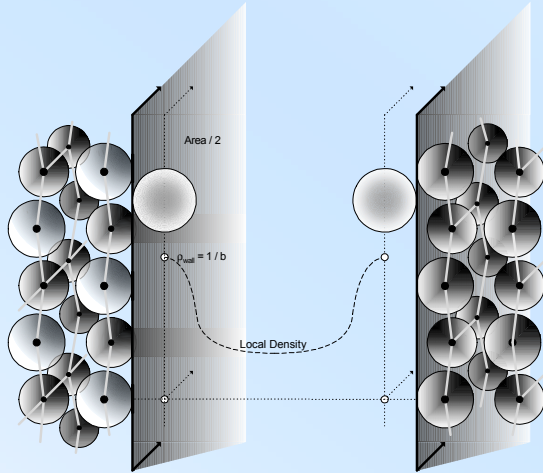


## **Conclusions: 2-D EOS**

- Mixing rules based on accessible-surface and on excess Gibbs free energy are effective in representing the binary mixture data within experimental uncertainties.
- These results suggest that:
  - Different adsorbates access different surface area on the same adsorbent
  - Mixture adsorption on the surface leads to non-random mixing

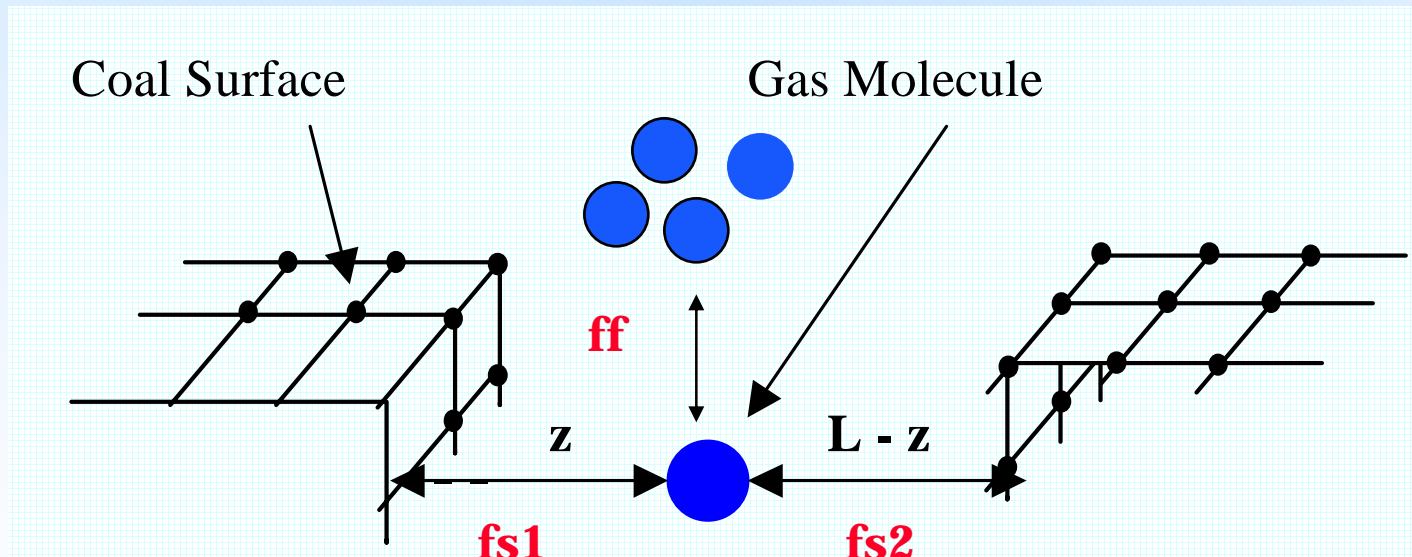


# **SLD-EOS Adsorption Mixture Modeling**



## The SLD-EOS Model

$$\mu_{fs}(z) = \mu_{fs1}(z) + \mu_{fs2}(L - z)$$



## *Extending SLD to Mixtures*

$$a = \sum_i \sum_j x_i x_j a_{ij}$$

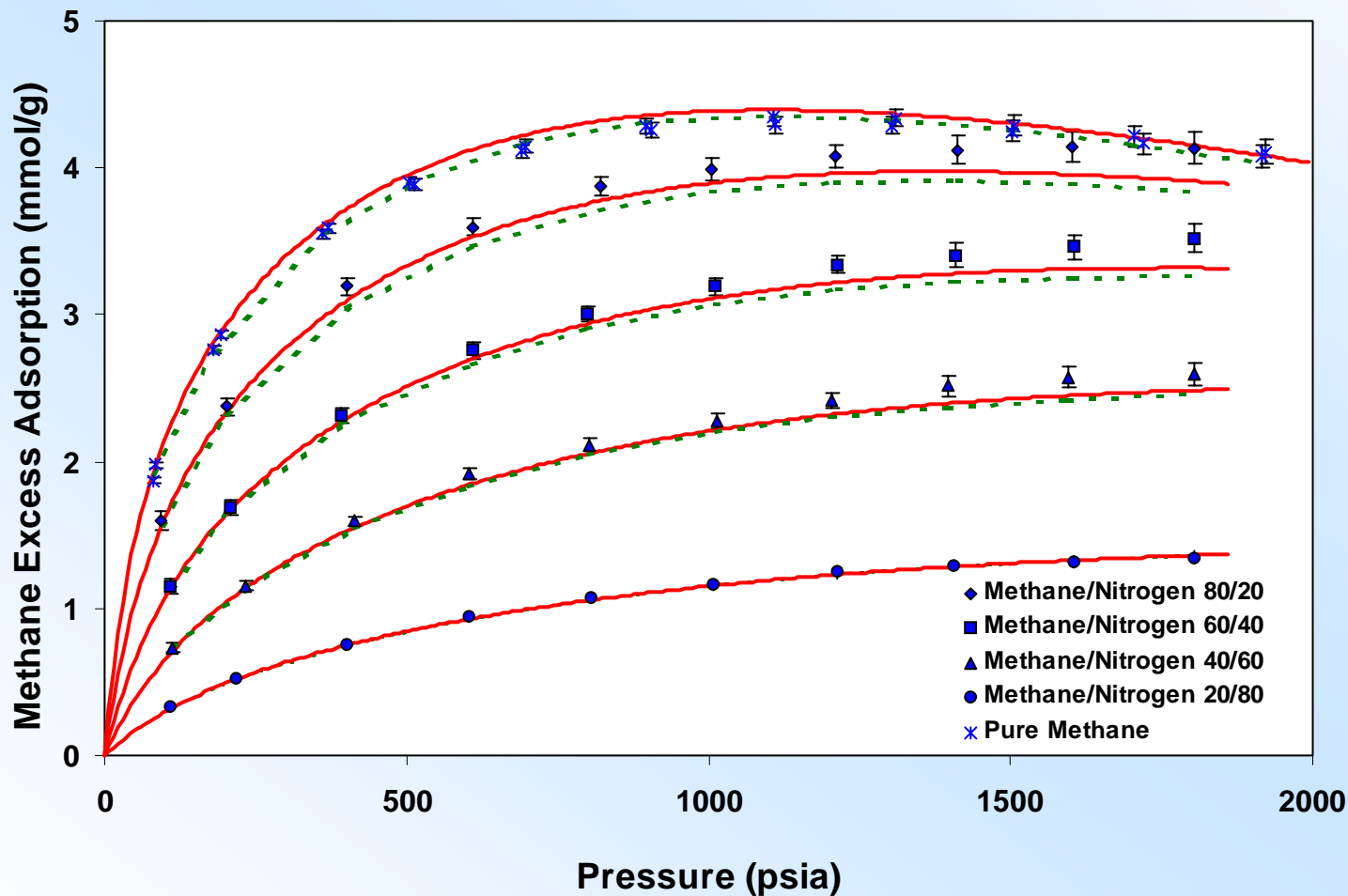
$$b = \sum_i \sum_j x_i x_j b_{ij}$$

$$a_{ij} = (a_i + a_j)(1 - C_{ij})/2$$

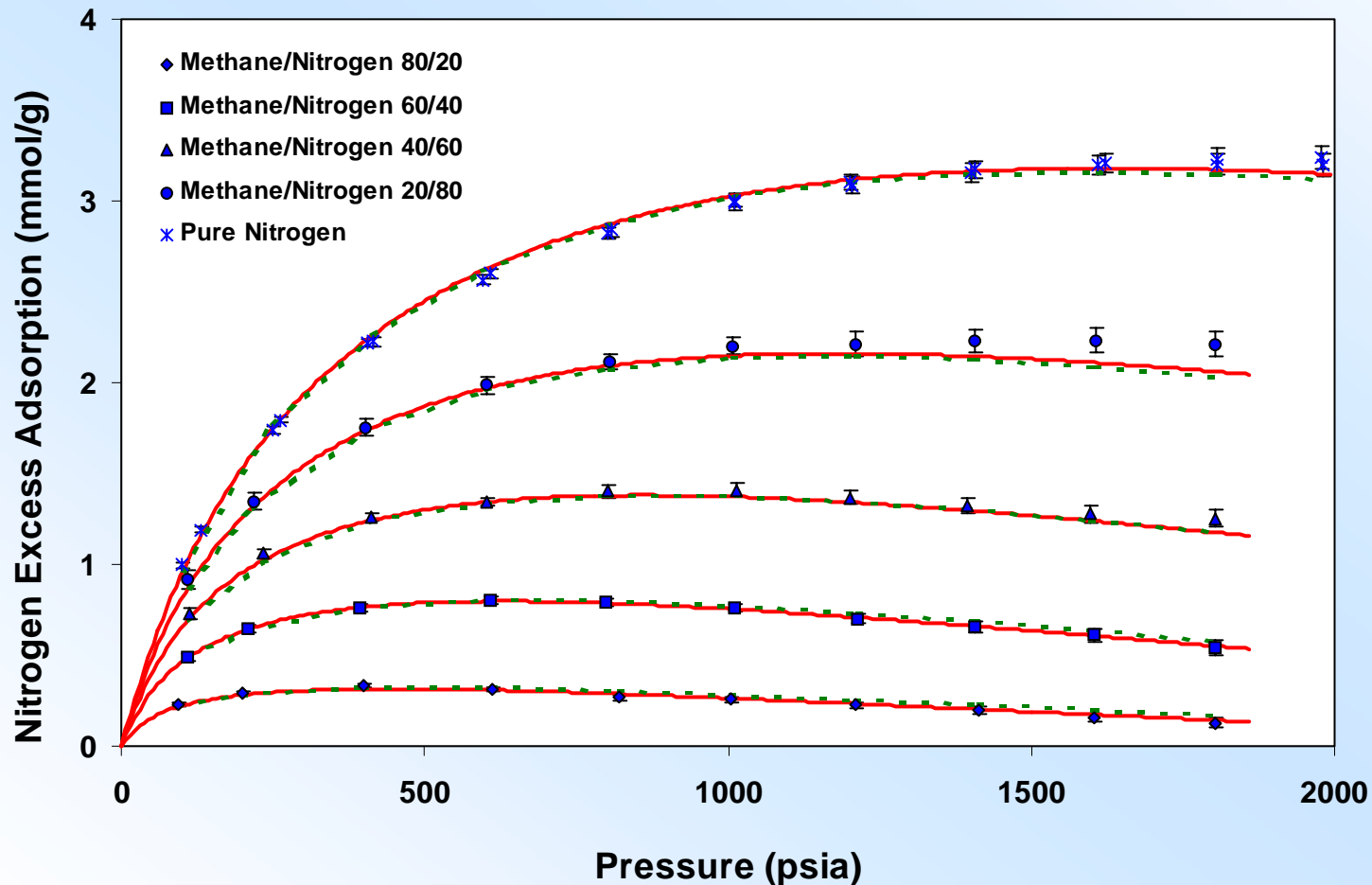
$$b_{ij} = (b_i + b_j)(1 + D_{ij})/2$$

- We set  $C_{ij}$  and  $D_{ij}$  to zero for all component interactions in the gas phase
- We regress  $C_{ij}$  ( $D_{ij}=0$ ) in the adsorbed phase

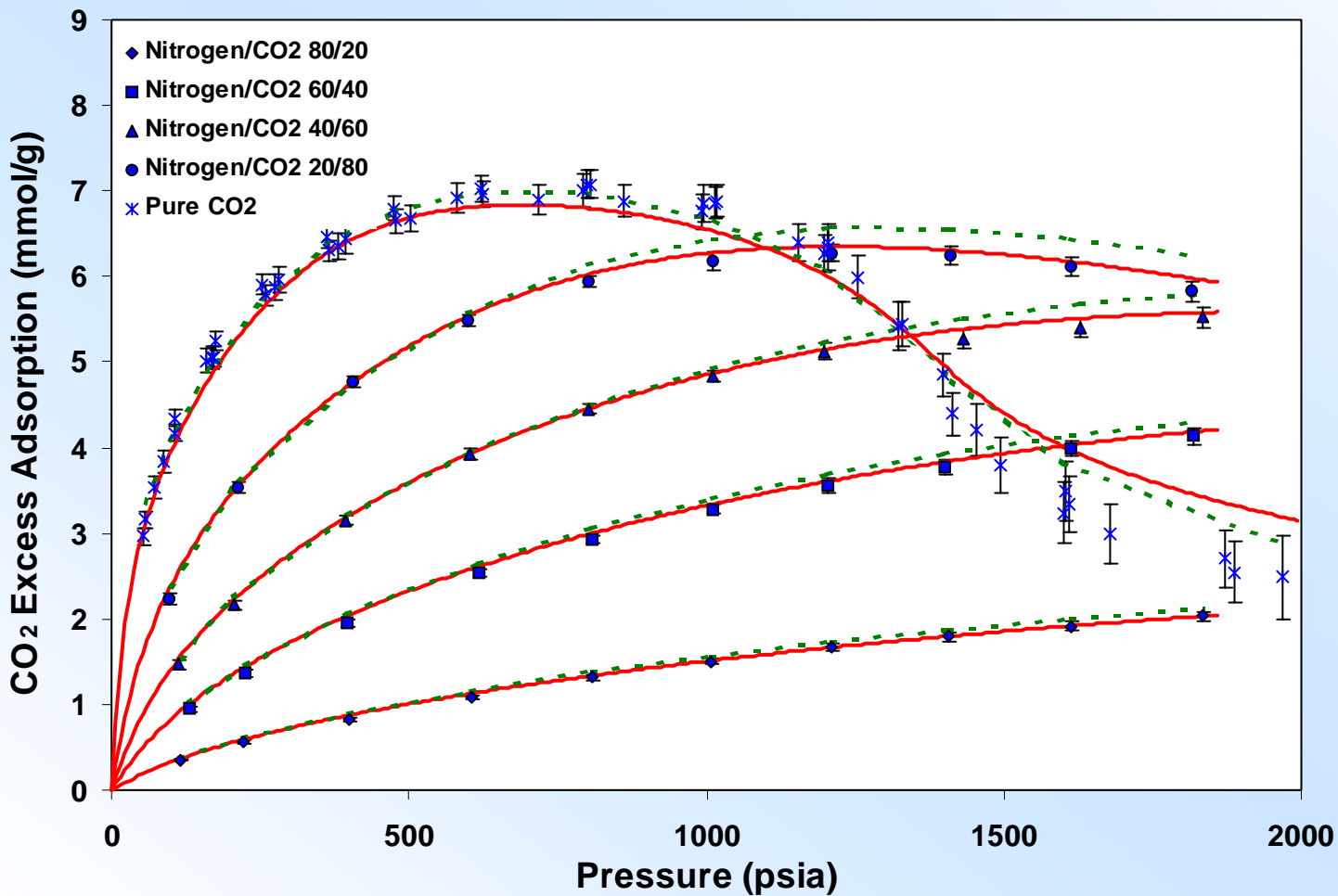
## Component Methane Adsorption in Methane/Nitrogen Mixtures



## Component Nitrogen Adsorption in Methane/Nitrogen Mixtures



## Component CO<sub>2</sub> Adsorption in Nitrogen/CO<sub>2</sub> Mixtures



## Conclusions: SLD-EOS

The EOS-SLD models:

- Correlate binary component adsorption isotherms of methane, nitrogen, and CO<sub>2</sub> mixtures within *expected experimental uncertainties*.
- Predict binary component adsorption isotherms for methane, nitrogen, and CO<sub>2</sub> mixtures, based on pure component isotherms, to within *twice experimental uncertainties on average*.

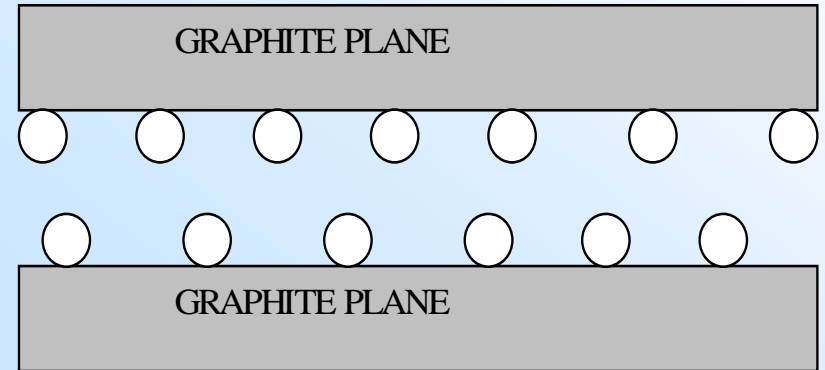
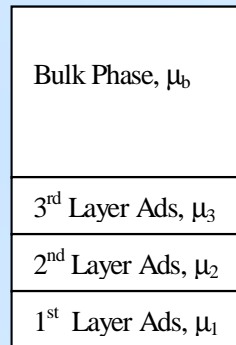
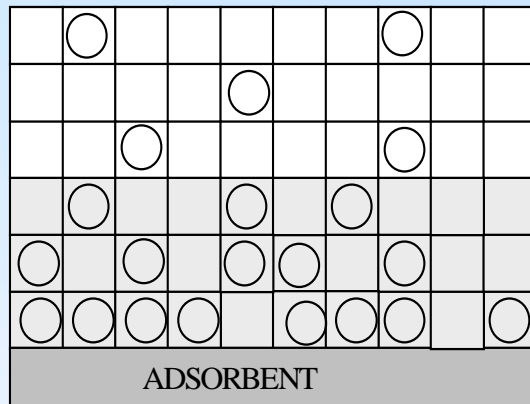


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*School of Chemical Engineering*

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# **OK Lattice Adsorption Mixture Modeling**

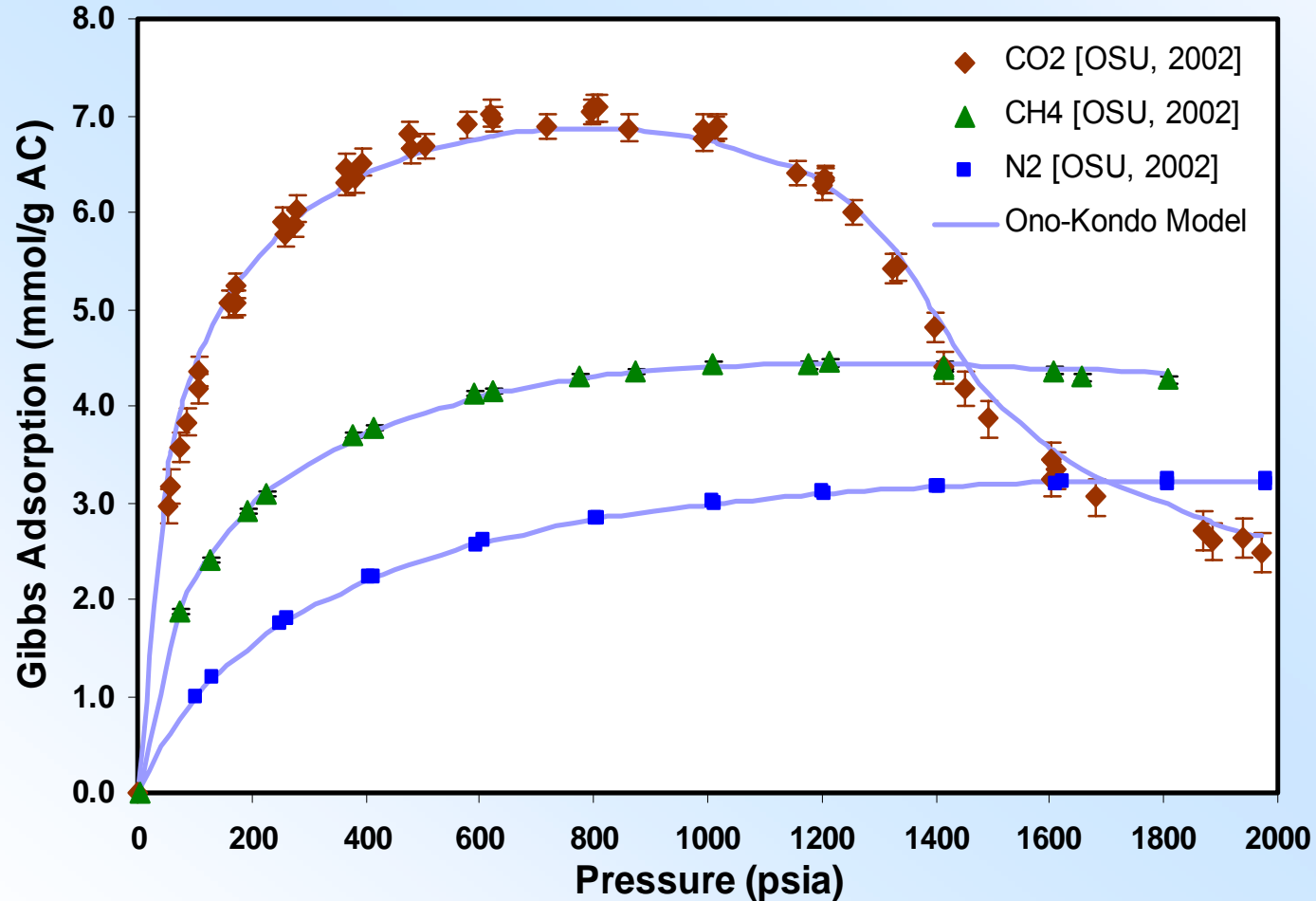
## OK Pure-Component Modeling



$$\ln \left[ \frac{x_{ads} (1 - x_b)}{x_b (1 - x_{ads})} \right] + (7x_{ads} - 8x_b) \epsilon / kT + \epsilon_s / kT = 0$$

$$\Gamma = 2C (x_{ads} - x_b) = 2C \left( \frac{\rho_{ads}}{\rho_{mc}} - \frac{\rho_b}{\rho_{mc}} \right)$$

## OK Regression Results for Pure Gases



## OK Lattice Model for Mixtures

### Equilibrium Equations:

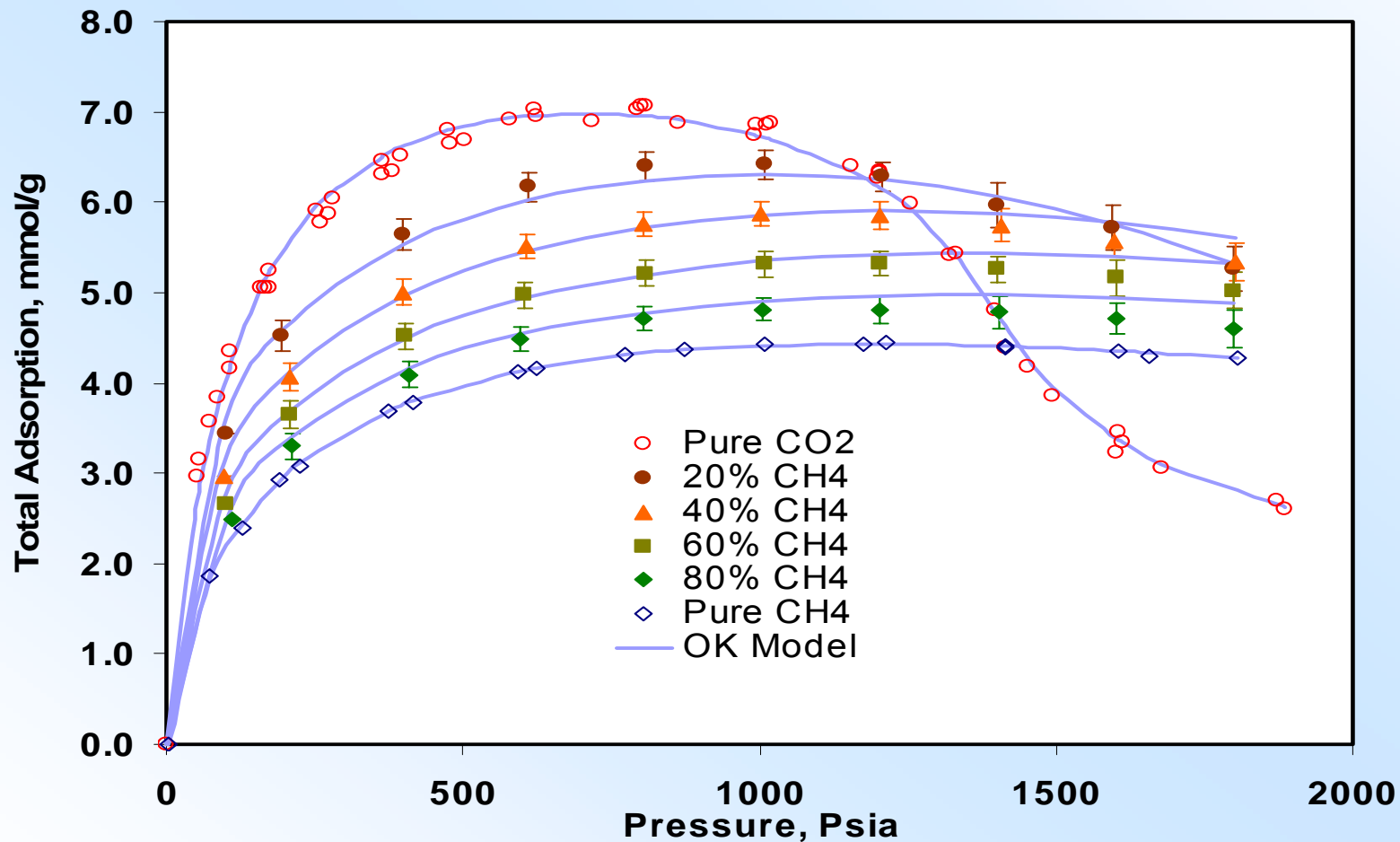
$$\ln \frac{x_A(1-x_{A,b}-x_{B,b})}{x_{A,b}(1-x_A-x_B)} + \frac{\epsilon_{AA}}{kT} ((z_1+1)x_A - z_0x_{A,b}) + \frac{\epsilon_{AB}}{kT} ((z_1+1)x_B - z_0x_{B,b}) + \frac{\epsilon_{As}}{kT} = 0$$

$$\ln \frac{x_B(1-x_{A,b}-x_{B,b})}{x_{B,b}(1-x_A-x_B)} + \frac{\epsilon_{BB}}{kT} ((z_1+1)x_B - z_0x_{B,b}) + \frac{\epsilon_{AB}}{kT} ((z_1+1)x_A - z_0x_{A,b}) + \frac{\epsilon_{Bs}}{kT} = 0$$

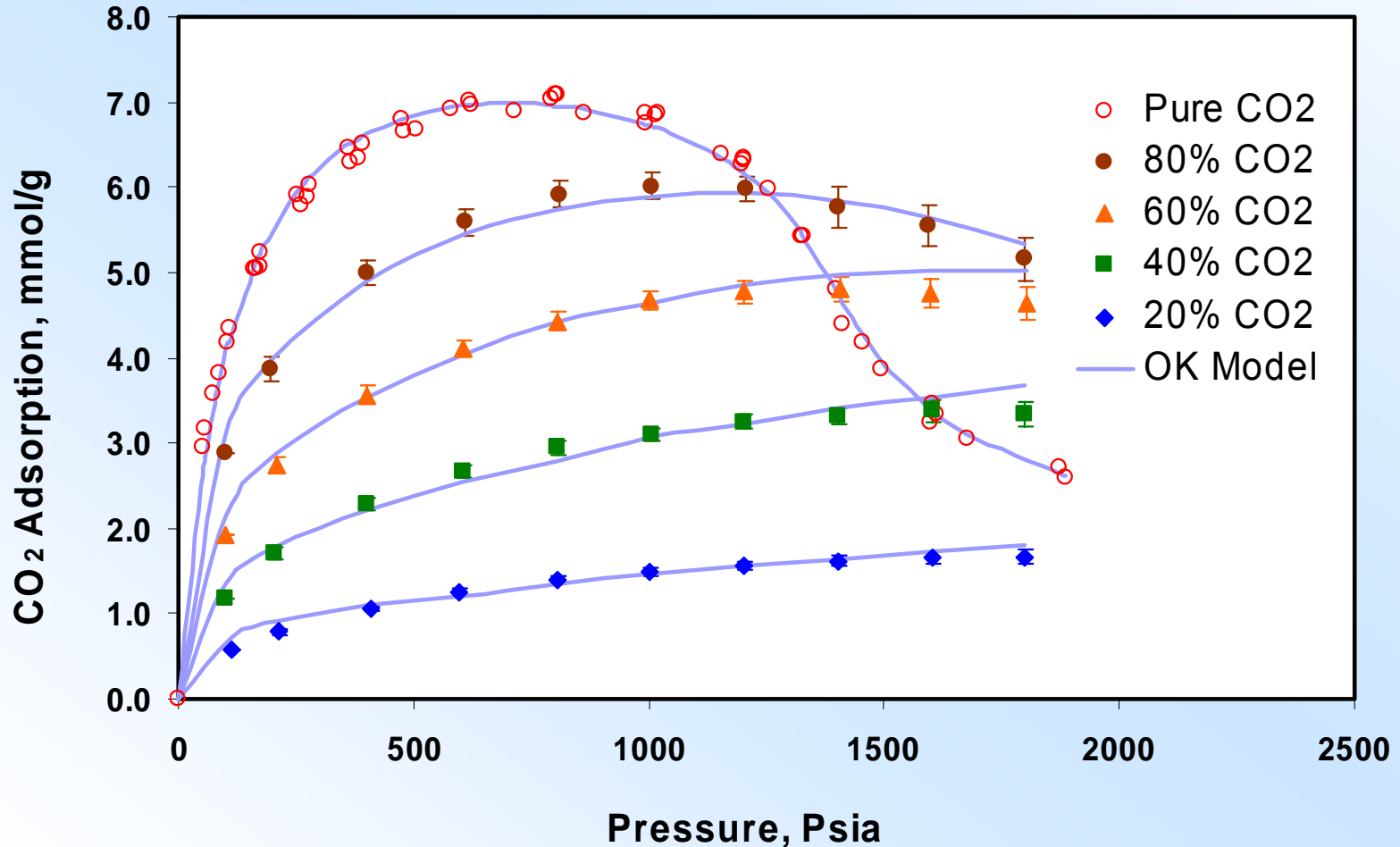
$$\epsilon_{AB} = (1 + C_{ij})(\epsilon_{AA} + \epsilon_{BB}) / 2$$

$$\Gamma_A = 2C_A(1 + S_{ij})(x_A - x_{A,b}) \quad \text{and} \quad \Gamma_B = 2C_B(1 - S_{ij})(x_B - x_{B,b})$$

## Total Gibbs Adsorption Isotherms



## Individual Component Gibbs Adsorption Isotherms



## **Conclusions: OK Model**

The extended OK model:

- Correlates the individual component adsorption in the binary systems within expected experimental uncertainties.
- Predicts total adsorption for the binary systems studied within twice the expected experimental uncertainties.

# ***Current Issues in Equilibrium CBM Adsorption Modeling***

## ***Fundamental Theory - OSU***

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## ***Matrix Characterization - PSU***

- Coal characterization 2003/4
- Coal-structure-based generalizations 2003/4

# Computational Algorithms for Equilibrium Adsorption Calculations

## Current Method

Solve for  $\omega$  and  $x_i$  by using experimental bulk phase compositions  $y_i$ .

At equilibrium, for each component i:

$$Z_a \hat{\phi}_i^a \omega_i = k_i \hat{\phi}_i^g y_i P, \quad \sum_i y_i = 1$$

Solve the above equations simultaneously to get  $\omega_i$ , then:

$$\omega = \sum_i \omega_i \quad x_i = \frac{\omega_i}{\omega}$$

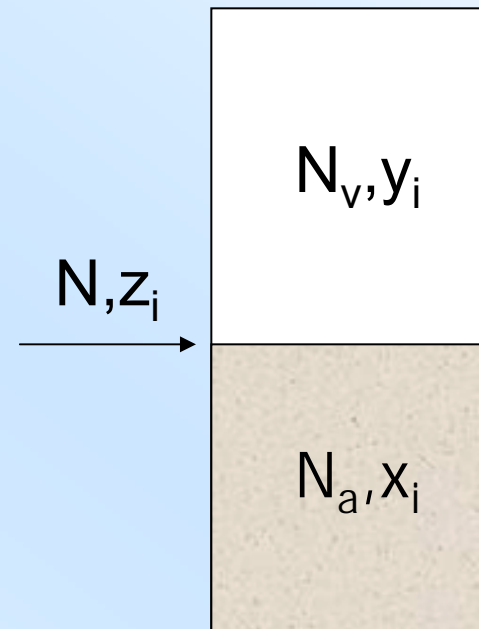
## Adsorption Iteration Function Algorithm

Solve for  $\omega$ ,  $x_i$ , and  $y_i$  by using feed composition  $z_i$

Mass Balance Equations:

$$F\left(\frac{N_a}{N}\right) = \sum_i \frac{(1-K_i)z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right)K_i} = 0$$

$$y_i = \frac{K_i z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right)K_i} \quad x_i = \frac{z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right)K_i}$$



## Algorithm 1

Initial guess:  $x_i, y_i, \omega \implies K_i = \frac{y_i}{x_i} = \frac{z_a \phi_i^a \omega}{k_i \phi_i^g P}$

$$F\left(\frac{N_a}{N}\right) = \sum_i \frac{(1-K_i)z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right)K_i} = 0$$

$$y_i = \frac{K_i z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right)K_i}$$

$$x_i = \frac{z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right)K_i}$$

$$\frac{N_a}{N} = \frac{\omega M_a}{\omega M_a + \rho_b V_b}$$

(Solve  $\omega$ )

## Algorithm 2

Initial guess:  $x_i, y_i, \omega$   $\longrightarrow$   $G_i = \frac{Z_a \hat{\phi}_i^a}{k_i \hat{\phi}_i^g P}$

$$F(\omega) = \sum_i \frac{(1 - G_i \omega) z_i}{\frac{\omega M_a}{\omega M_a + \rho_b V_b} + \left(1 - \frac{\omega M_a}{\omega M_a + \rho_b V_b}\right) G_i \omega} = 0$$

$$\frac{N_a}{N} = \frac{\omega M_a}{\omega M_a + \rho_b V_b} \longrightarrow$$

$$y_i = \frac{G_i \omega z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right) G_i \omega}$$

$$x_i = \frac{z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right) G_i \omega}$$

## Algorithm 3

Initial guess:  $x_i, y_i$

$$F(\omega) = \sum_i \frac{(1 - \frac{Z_a \hat{\phi}_i^a}{k_i \hat{\phi}_i^g P} \omega) z_i}{\frac{\omega M_a}{\omega M_a + \rho_b V_b} + \left(1 - \frac{\omega M_a}{\omega M_a + \rho_b V_b}\right) \frac{Z_a \hat{\phi}_i^a}{k_i \hat{\phi}_i^g P} \omega} = 0$$

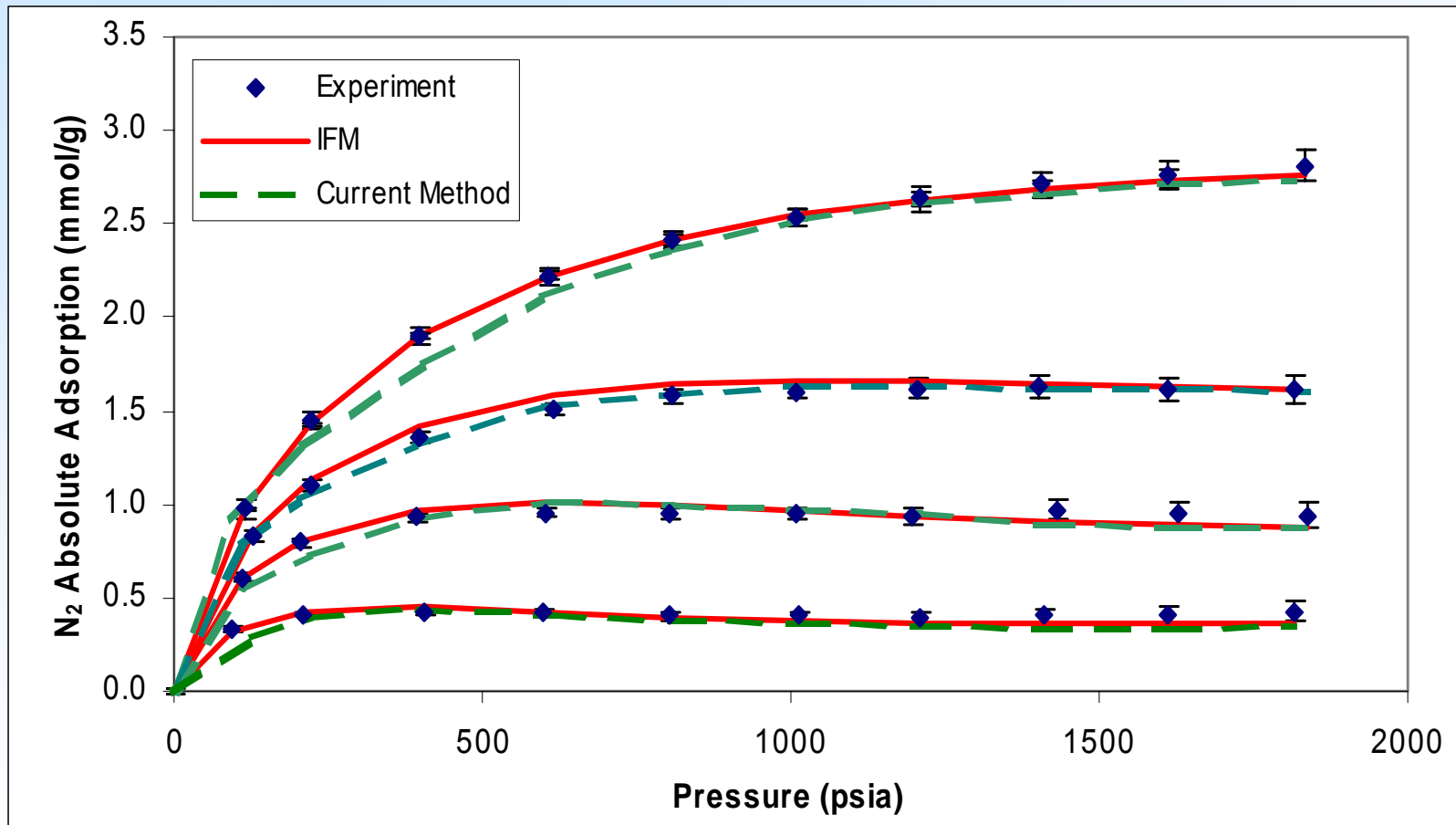
$$\frac{N_a}{N} = \frac{\omega M_a}{\omega M_a + \rho_b V_b}$$

$$G_i = \frac{Z_a \hat{\phi}_i^a}{k_i \hat{\phi}_i^g P}$$

$$y_i = \frac{G_i \omega z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right) G_i \omega}$$

$$x_i = \frac{z_i}{\frac{N_a}{N} + \left(1 - \frac{N_a}{N}\right) G_i \omega}$$

# Results: $N_2+CO_2$ on Activated Carbon at 113 °F



## ***Conclusions: Algorithms***

- The proposed adsorption IFM algorithm is robust
- The proposed flash calculations produce consistent computational results

# ***Current Issues in Equilibrium CBM Adsorption Modeling***

## ***Fundamental Theory - OSU***

- Improved mixture modeling 2002
- Computational algorithms 2002
- Effect of moisture on adsorption behavior 2003
- Model parameter generalizations 2003

## ***Matrix Characterization - PSU***

- Coal characterization 2003/4
- Coal-structure-based generalizations 2003/4

## *Approach*

- Investigate the effect of each factor on our ability to simulate accurately CBM production and CO<sub>2</sub> sequestration processes
- Assess the potential economic impact of the uncertainty associated with each factor
- Rank order the issues to be resolved
- Identify the required resources for resolving the priority issues
- Develop research plans to resolve the priority issues

## Moisture Effects

### Current Working Assumption

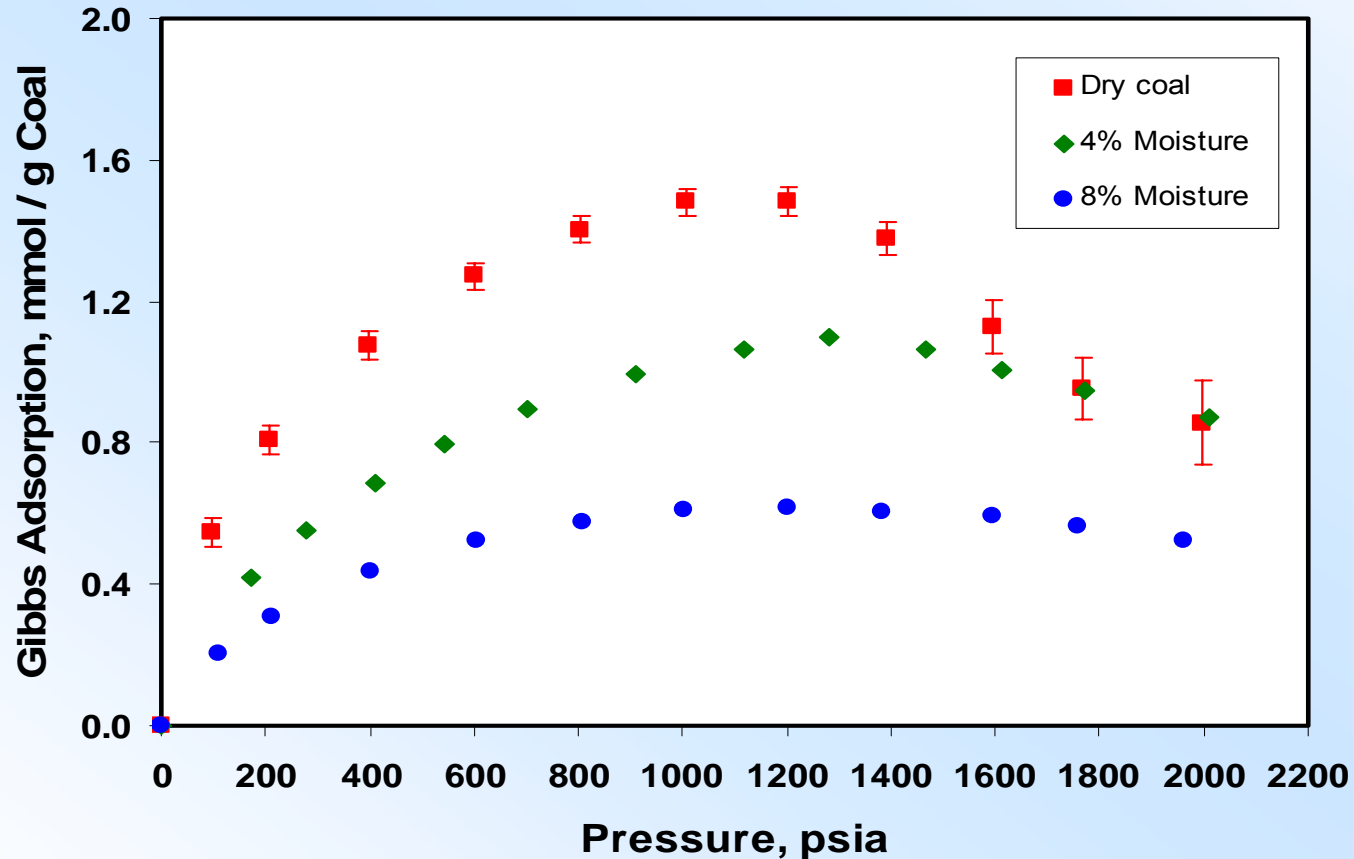
*Operations occur in super-saturated water conditions; therefore, adsorption isotherms involving super-moist adsorbents are sufficient for the purpose.*

### In Reality

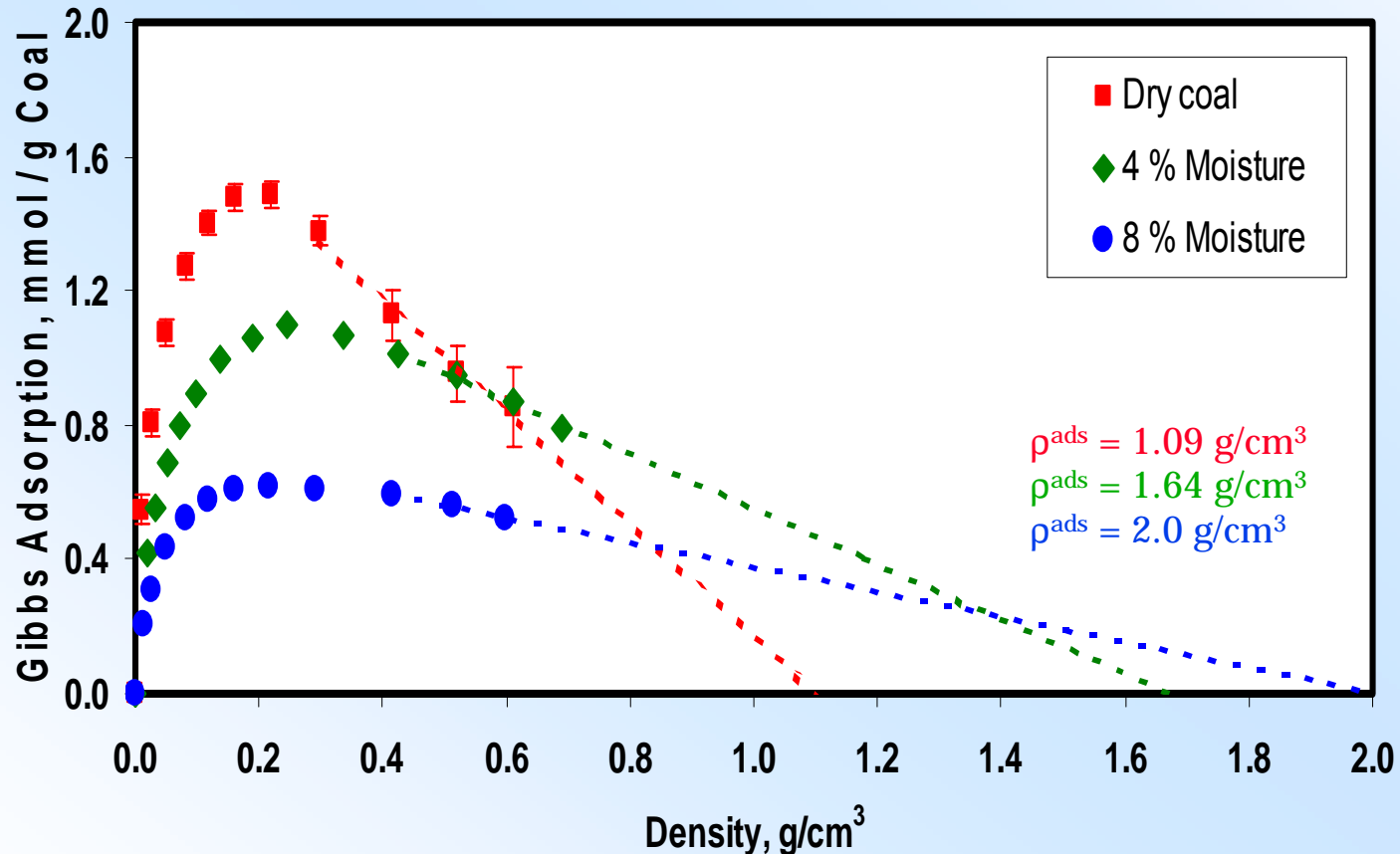
- Moisture content may affect significantly the:
  - Adsorption capacity
  - Adsorbed-phase density
  - Mixture adsorption behavior
  - Data interpretation and reconciliation
- The current accounting for moisture effects is not adequate

*We need more rigorous modeling of wet adsorption behavior!*

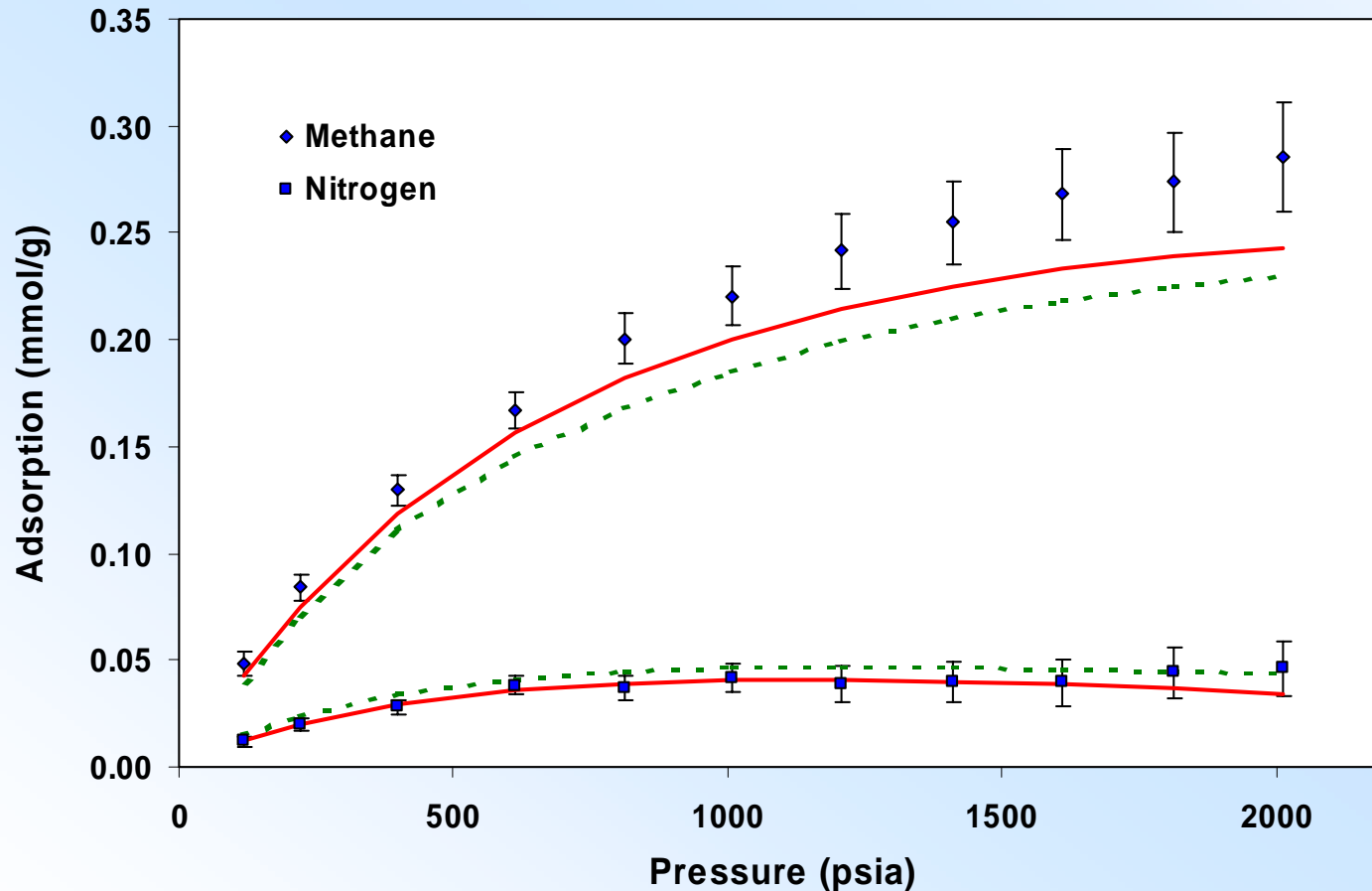
## Moisture Effect on CO<sub>2</sub> Adsorption Isotherms: Illinois-6 at 131 °F



## Moisture Effect on CO<sub>2</sub> Adsorbed Phase Density: Illinois-6 at 131 °F



## SLD Modeling of Wet Methane/Nitrogen Adsorption on Tiffany Coal at 131 °F



## Moisture Effects: Case 1

- **Assumptions**

No standing water, no salts

- **Approach**

- Treat the situation as an **AD/Gas** equilibrium problem
- Develop an adsorption model to cover the full pTx diagram for CBM gas/water systems

**Wet Gas**

$\text{CO}_2/\text{H}_2\text{O}$

**Wet Coal**

$\text{CO}_2/\text{H}_2\text{O}$

## Moisture Effects: Case 2

- Assumptions**

Standing water, no salts

- Approach 1**

- Treat the situation as an **AD/Liq/Gas** equilibrium problem
- Develop an accurate 3D EOS for CBM gas/water systems
- Develop an adsorption model to cover the full pTx diagram for CBM gas/water systems

Wet Gas

CO<sub>2</sub>/H<sub>2</sub>O

Standing Water

CO<sub>2</sub>/H<sub>2</sub>O

Wet Coal

CO<sub>2</sub>/H<sub>2</sub>O

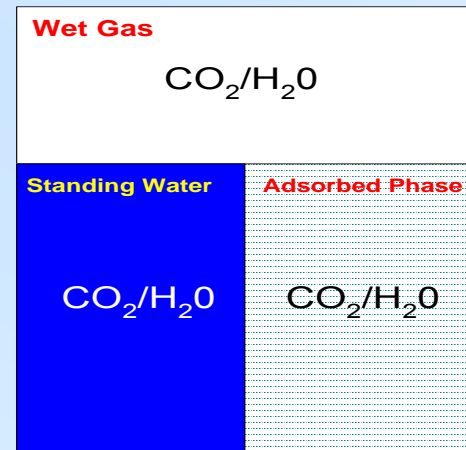
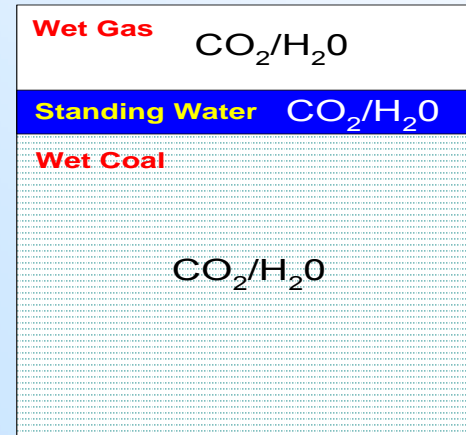
## Moisture Effects: Case 2 - continued

- Assumptions**

Standing water, no salts

- Approach 2**

- Develop an accurate EOS for CBM gas/water systems
- Conduct a VLE calculation for CBM gas/water system
- Use the VLE gas composition to conduct **AD/Gas** calculation for the gas mixture
- Repeat the procedure until component fugacities are equal across the three phases



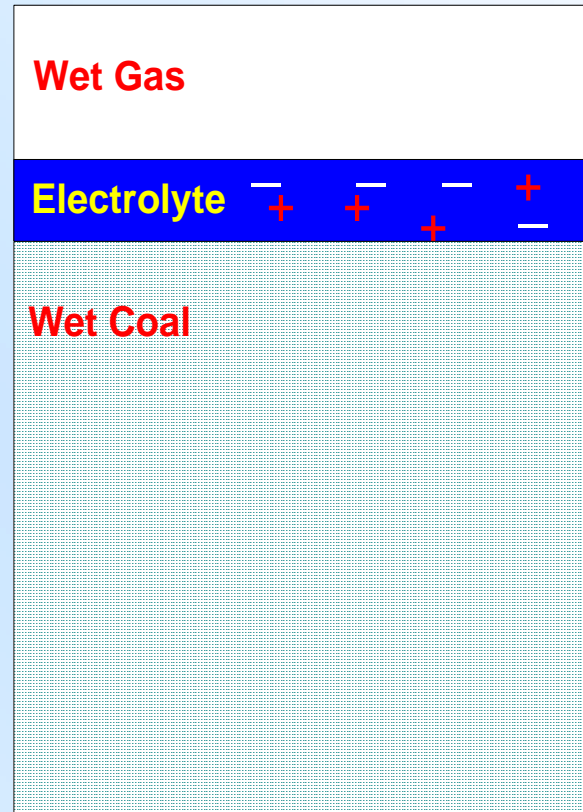
## Moisture Effects: Case 3

- Assumptions**

Standing water, with salts

- Approach 3**

- Develop an accurate 3D EOS for CBM gas/water systems
- Conduct a **electrolyte** VLE calculations for gas/water system
- Use the VLE gas composition to conduct **AD/Gas** calculations for the gas mixture
- Repeat the procedure until component fugacities are equal in the three phases



## ***Moisture Effects: Experimental Measurements***

- **Apparatus**

Install density meter for increased accuracy of mixture data

- **Measurements**

Conduct equilibrium adsorption measurements involving

- a) the targeted gases
- b) on activated carbon and two well characterized coals
- c) for at least three levels of moisture content

## Closure

### ***Fundamental Theory - OSU***

- Improved mixture modeling *done*
- Computational algorithms *done*
- Effect of moisture on adsorption behavior 2003
- Model parameter generalizations 2003

### ***Matrix Characterization - PSU***

- Coal characterization 2003/4
- Coal structure-based generalizations 2003/4